Further Maths Bridging Unit

Welcome to fascinating and challenging subject of Further Mathematics A-level. You're clearly not satisfied with one maths A-level and would like to double the dose! Further Maths will build on much of the work in standard maths and develop it to a far more advanced level as well as building a bridge to university level study.

It is worth stating here that Further Maths is a ***very challenging*** course. The difficulty of the content and depth of understanding required is worlds apart from GCSE. You will be expected to have a highly driven interest in the subject as well as a hardworking attitude, and to be proactive in working with others and seeking help when needed.

Below is the bridging unit work to help best prepare you for September. The first section comprises of 10 short questions while the second section comprises of 5 longer questions. You are expected to work through these over the summer and hand in your work at the beginning of Year 12. The more effort and thought you put into this, the better prepared you will be.

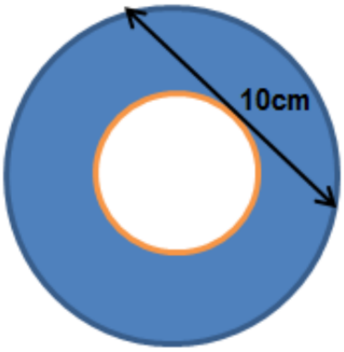
You are also encouraged to read around the subject and expand your maths knowledge beyond the curriculum. You can check out the department reading recommendations or feel free to find a book of your choice.

Have a fantastic summer and we look forward to seeing you in September!

NWS Mathematics Department

[Short problems](#)

Can you work out the shaded area in the diagram (the line shown just touches the smaller circle)?



Find the value of

$$\frac{99}{100} \times \frac{80}{81} \times \frac{63}{64} \times \frac{48}{49} \times \frac{35}{36} \times \frac{24}{25} \times \frac{15}{16} \times \frac{8}{9} \times \frac{3}{4}.$$

Write your answer in the form $\frac{a}{b}$, where a and b are positive integers with no common factors other than 1.

A point E lies outside the rectangle $ABCD$ such that CBE is an equilateral triangle. The area of the pentagon $ABECD$ is five times the area of the triangle CBE.

What is the ratio of the lengths $AB : AD$?

Write your answer in the form $a : 1$.

A sequence is defined as follows:

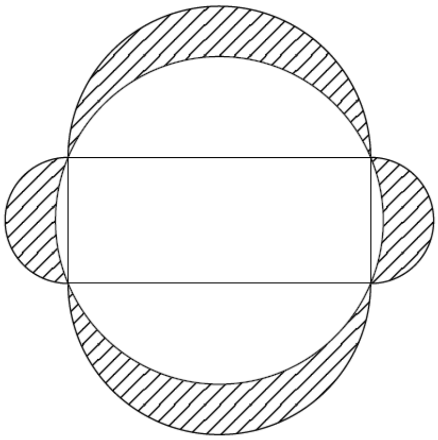
$u_1 = 123$.

For $n \geq 1$, define u_{n+1} = the sum of the squares of the digits of u_n.

For example, $u_2 = 1^2 + 2^2 + 3^2 = 14$, $u_3 = 1^2 + 4^2 = 17$.

What is the value of u_{100}?

Four semicircles are drawn on the sides of a rectangle with width 10 cm and length 24 cm. A circle is drawn that passes through the four vertices of the rectangle.



What is the value, in cm^2, of the shaded area?

Alfred, Brenda, Colin, David and Erica have to sit on a row of five chairs. Alfred does not want to sit next to Brenda. David does not want to sit next to Erica.

In how many ways can these five people arrange themselves and ensure the above conditions are met?

(a) Which positive integer in the range from 1 to 250 has more different prime divisors than any other integer in this range?

(b) When $n = 5$ the product $n(n+1)(n+2)$ can be written as the product of four distinct primes. Indeed, when $n = 5$

$$n(n+1)(n+2) = 5 \times 6 \times 7 = 2 \times 3 \times 5 \times 7.$$

What is the least positive integer n such that $n(n+1)(n+2)$ can be written as a product of *five* distinct primes?

Find the value of

$$\left(\left(2^{\frac{3}{4}}+1\right)^2+\left(2^{\frac{3}{4}}-1\right)^2\right)\left(\left(2^{\frac{3}{4}}+1\right)^2+\left(2^{\frac{3}{4}}-1\right)^2-2^2\right).$$

The points $A(1,2)$ and $B(-2,1)$ are two vertices of a rectangle $ABCD$. The diagonal CA produced passes through the point $(2,9)$. Calculate the coordinates of the vertices C and D.

The inequalities $x^2+3x+2>0$ and $x^2+x<2$ are met by all x in the region:

a) $x<2$; b) $-1<x<1$; c) $x>-1$; d) $x>-2$

Answers

1. 25π

2. $\frac{11}{20}$

3. $\sqrt{3} : 1$

4. 4

5. 240

6. 48

7. (a) 210

   (b) 13

8. 28

9. C is $(0, -5)$ and D is $(3, -4)$

10. $-1 < x < 1$

Longer problems

The following five problems are designed to get you to think in a little bit more depth about some mathematical concepts that you have met before. These problems will take you a little bit longer than the short ones and will require a more detailed solution. In some cases, they are quite open-ended.

You can attempt these problems in any order of your choice. The solutions are linked at the end but please make sure you make a proper attempt before looking at them. There is very little to be gained from just reading through an answer without grappling with the problem first!

Problem 1: A tangent is…

You will be familiar with what a tangent is from GCSE. Tangents are used a lot in A level Maths because they represent the rate of change of a function. This all comes under the umbrella of calculus, which you will begin to study in Year 12.

The following problem is designed to get you to think about what a tangent is and what they look like. By working through this problem, you will improve your understanding of tangents, which will aid your progress with studying calculus.

Problem 1: A tangent is…

Here are some suggestions of how we could define a tangent to a curve at a point.

1) "A tangent is a straight line which only meets the curve at that one point."
2) "A tangent is a straight line which touches the curve at that point only."
3) "A tangent is a straight line which meets the curve at that point, but the curve is all on one side of the line."
4) "A tangent is a straight line which meets the curve at that point, but near that point, the curve is all on one side of the line."

Which, if any, of these proposed definitions works for a circle?

For each one, can you find or sketch an example to show that the proposed definition does not work for all curves?

Can you come up with a better definition?

Problem 2: Scary Sum

At GCSE, you will have learned how to manipulate surds. The following problem is designed to get you to think about how to manipulate an expression involving surds ***without*** using a calculator.

Problem 2: Scary Sum

Evaluate the sum

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{15}+\sqrt{16}}$$

(You might want to use a calculator to get an estimate of the answer, but in order to get the exact answer you will have to do it by hand!)

Can you find a similar sum that evaluates to 5?

Can you find a similar sum that evaluates to a number that is not an integer?

Problem 3: Parabella

Coordinate geometry is a massive topic in A level Maths and Further Maths. The skills you have learned at GCSE will be developed further and applied to a variety of problems.

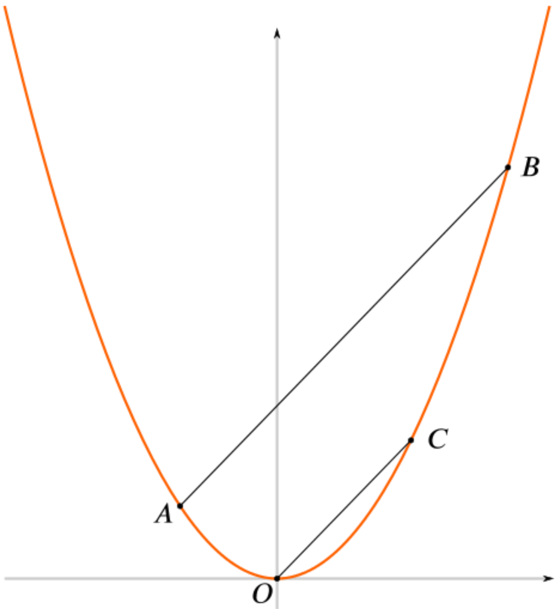
Problem 3: Parabella

Take any two points A and B on the parabola $y = x^2$.

Draw the line OC through the origin, parallel to AB, cutting the parabola again at C.

Let A have coordinates (a, a^2), let B have coordinates (b, b^2), and let C have coordinates (c, c^2).

Prove that $a + b = c$.



Imagine drawing another parallel line DE, where D and E are two other points on the parabola. Extend the ideas of the previous result to prove that the midpoints of each of the three parallel lines lie on a straight line.

Problem 4: Powerful Quadratics

You will be familiar with a number of ways of solving quadratic equations. This problem requires you to inspect an equation and to think about possible values that could satisfy it using your knowledge of indices.

There are more solutions than you might think…

Problem 4: Powerful Quadratics

(i) Find all real solutions of the equation

$$(x^2 - 7x + 11)^{(x^2 - 11x + 30)} = 1$$

(ii) Find all real solutions of the equation

$$(2 - x^2)^{(x^2 - 3\sqrt{2}x + 4)} = 1$$

Problem 5: Two roots differ by 5

Solving equations is an important skill throughout mathematics, but in Further Maths, we begin to think in more depth about the relationship between the roots of an equation.

This problem is designed to get you to think about how quadratic and cubic equations are formed from their roots. This idea will be developed considerably in the first term of Further Maths as we consider roots of polynomials and begin to discover complex numbers.

Problem 5: Two roots differ by 5

(i) The equation $x^3 + ax + b = 0$ is satisfied by the values $x = 1$ and $x = 2$. Find the values of a and b; and find also the third value of x.

(ii) The equation $x^2 - 12x + k = 0$ is satisfied by two values of x that differ by 5. Find these two values of x and the value of k.

Solutions

These problems were all taken from the 'Underground Maths' project, where you can find lots of enrichment material for A-level Maths and Further Maths. The solutions are linked below:

Solution 1: A tangent is…

Solution 2: Scary Sum

Solution 3: Parabella

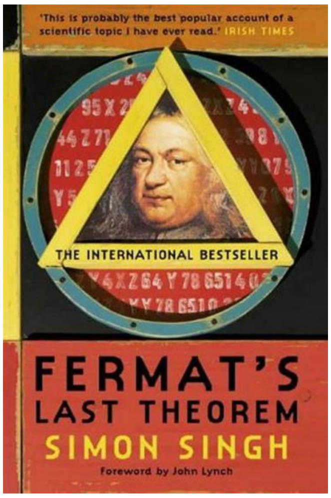
Solution 4: Powerful Quadratics

Solution 5: Two roots differ by 5

Maths Department Reading Recommendations

Easier reads

Fermat's Last Theorem (Simon Singh)

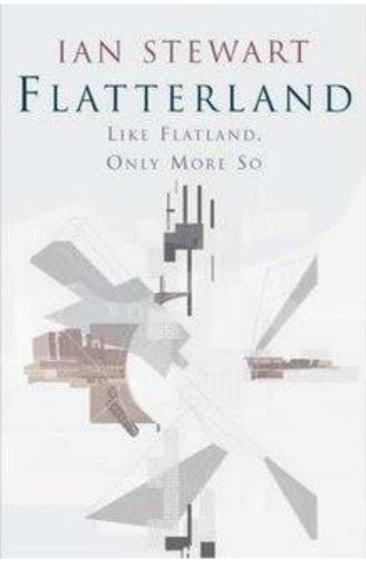


'I have a truly marvellous demonstration of this proposition which this margin is too narrow to contain.'

It was with these words, written in the 1630s, that Pierre de Fermat intrigued and infuriated the mathematics community. For over 350 years, proving Fermat's Last Theorem was the most notorious unsolved mathematical problem, a puzzle whose basics most children could grasp but whose solution eluded the greatest minds in the world. In 1993, after years of secret toil, Englishman Andrew Wiles announced to an astounded audience that he had cracked Fermat's Last Theorem. He had no idea of the nightmare that lay ahead.

In 'Fermat's Last Theorem' Simon Singh has crafted a remarkable tale of intellectual endeavour spanning three centuries, and a moving testament to the obsession, sacrifice and extraordinary determination of Andrew Wiles: one man against all the odds.

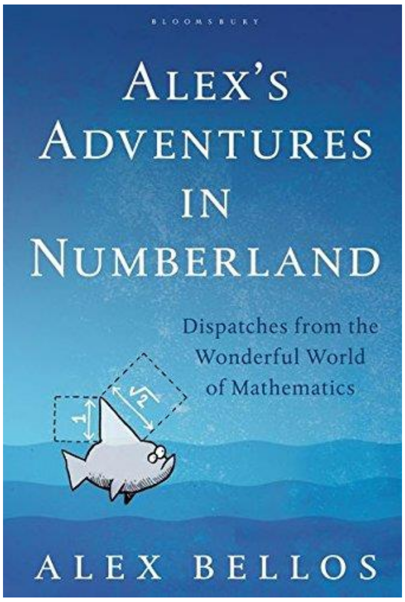
Flatterland (Ian Stewart)



Through larger-than-life characters and an inspired story line, Flatterland explores our present understanding of the shape and origins of the universe, the nature of space, time, and matter, as well as modern geometries and their applications.

The journey begins when our heroine, Victoria Line, comes upon her great-great-grandfather A. Square's diary, hidden in the attic. The writings help her to contact the Space Hopper, who tempts her away from her home and family in Flatland and becomes her guide and mentor through ten dimensions. In the tradition of Alice in Wonderland and The Phantom Toll Booth, this magnificent investigation into the nature of reality is destined to become a modern classic.

Alex's Adventures in Numberland (Alex Bellos)



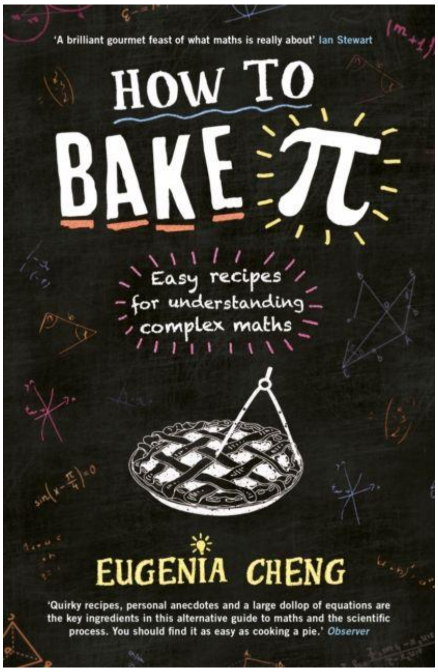
The world of maths can seem mind-boggling, irrelevant and, let's face it, boring. This groundbreaking book reclaims maths from the geeks.

Mathematical ideas underpin just about everything in our lives: from the surprising geometry of the 50p piece to how probability can help you win in any casino. In search of weird and wonderful mathematical phenomena, Alex Bellos travels across the globe and meets the world's fastest mental calculators in Germany and a startlingly numerate chimpanzee in Japan.

Packed with fascinating, eye-opening anecdotes, Alex's Adventures in Numberland is an exhilarating cocktail of history, reportage and mathematical proofs that will leave you awestruck.

More in-depth reads

How to Bake Pi (Eugenia Cheng)

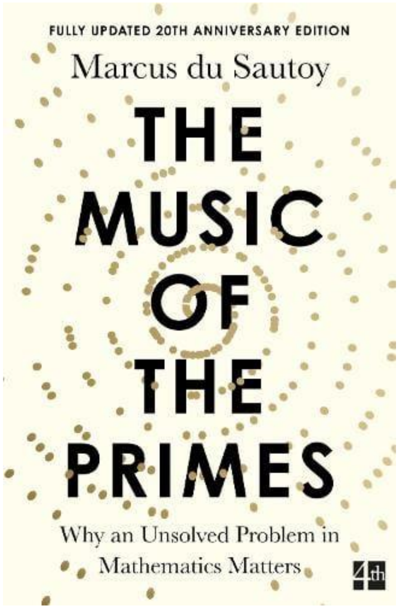


What is math? How exactly does it work? And what do three siblings trying to share a cake have to do with it?

In How to Bake Pi, math professor Eugenia Cheng provides an accessible introduction to the logic and beauty of mathematics, powered, unexpectedly, by insights from the kitchen. We learn how the bechamel in a lasagna can be a lot like the number five, and why making a good custard proves that math is easy, but life is hard. At the heart of it all is Cheng's work on category theory, a cutting-edge "mathematics of mathematics," that is about figuring out how math works.

Combined with her infectious enthusiasm for cooking and true zest for life, Cheng's perspective on math is a funny journey through a vast territory no popular book on math has explored before. So, what is math? Let's look for the answer in the kitchen.

The Music of the Primes (Marcus Du Sautoy)

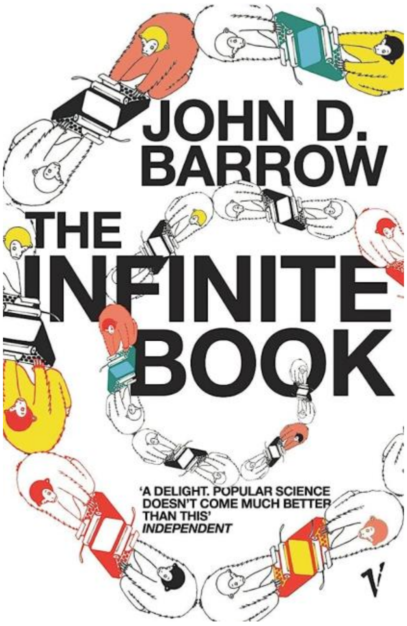


Prime numbers are the very atoms of arithmetic. They also embody one of the most tantalising enigmas in the pursuit of human knowledge. How can one predict when the next prime number will occur? Is there a formula which could generate primes? These apparently simple questions have confounded mathematicians ever since the Ancient Greeks.

In 1859, the brilliant German mathematician Bernard Riemann put forward an idea which finally seemed to reveal a magical harmony at work in the numerical landscape. Yet Riemann, a hypochondriac and a troubled perfectionist, never publicly provided a proof for his hypothesis and his housekeeper burnt all his personal papers on his death.

Whoever cracks Riemann's hypothesis will go down in history, for it has implications far beyond mathematics. In science, it has critical ramifications in Quantum Mechanics, Chaos Theory, and the future of computing. Pioneers in each of these fields are racing to crack the code and a prize of $1 million has been offered to the winner. As yet, it remains unsolved.

The Infinite Book (John D. Barrow)



Everything you might want to know about infinity - in history and all the way to today's cutting-edge science.

Infinity is surely the strangest idea that humans have ever had. Where did it come from and what is it telling us about our Universe? Can there actually be infinities? Can you do an infinite number of things in a finite amount of time? Is the Universe infinite?

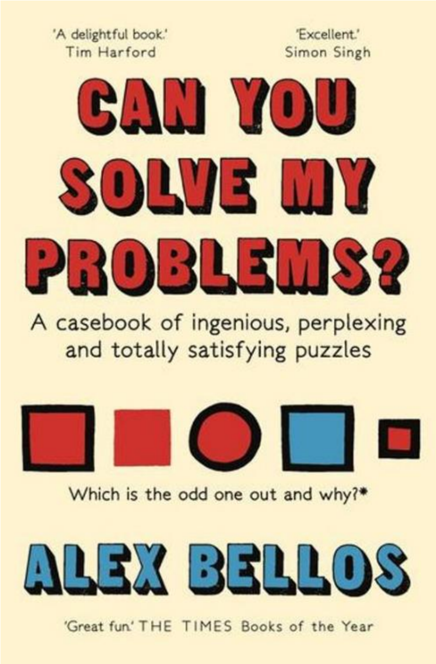
Infinity is also the place where things happen that don't. What is it like to live in a Universe where nothing is original, where you can live forever, where anything that can be done, is done, over and over again?

These are some of the deep questions that the idea of the infinite pushes us to ask. Throughout history, the infinite has been a dangerous concept. Many have lost their lives, their careers, or their freedom for talking about it.

The Infinite Book will take you on a tour of these dangerous questions and the strange answers that scientists, mathematicians, philosophers and theologians have come up with to deal with its threats to our sanity.

Puzzle books

Can You Solve My Problems? (Alex Bellos)



Are you smarter than a Singaporean ten-year-old?

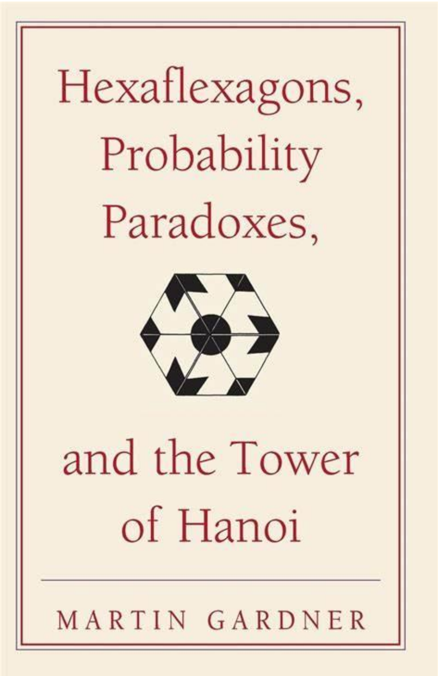
Can you beat Sherlock Holmes?

If you think the answer is yes – I challenge you to solve my problems.

Here are 125 of the world's best brainteasers from the last two millennia, taking us from ancient China to medieval Europe, Victorian England to modern-day Japan, with stories of espionage, mathematical breakthroughs and puzzling rivalries along the way.

Pit your wits against logic puzzles and kinship riddles, pangrams and river-crossing conundrums. Some solutions rely on a touch of cunning, others call for creativity, others need mercilessly logical thought. Some can only be solved be 2 per cent of the population. All are guaranteed to sharpen your mind. Let's get puzzling!

Hexaflexagons, Probability, Paradoxes, and the Tower of Hanoi (Martin Gardner)



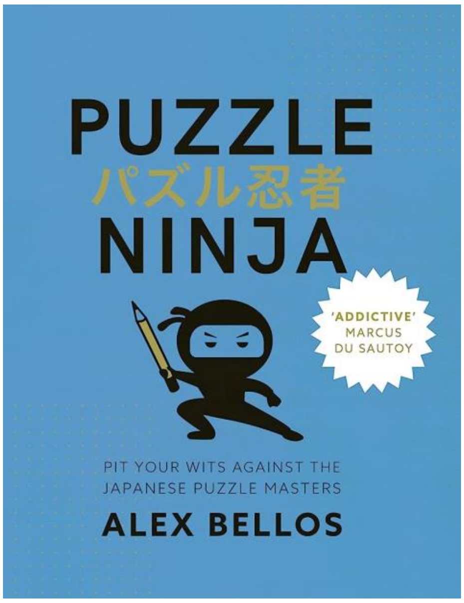
Paradoxes and paper-folding, Moebius variations and mnemonics, fallacies, magic squares, topological curiosities, parlor tricks, and games ancient and modern, from Polyominoes, Nim, Hex, and the Tower of Hanoi to four-dimensional ticktacktoe.

These mathematical recreations, clearly and cleverly presented by Martin Gardner, delight and perplex while demonstrating principles of logic, probability, geometry, and other fields of mathematics.

This book of the earliest of Gardner's enormously popular Scientific American columns and puzzles continues to challenge and fascinate readers.

Now the author, in consultation with experts, has added updates to all the chapters, including new game variations, mathematical proofs, and other developments and discoveries.

Puzzle Ninja (Alex Bellos)



Puzzles are so enjoyable. They get your brain sparking and the competitive spirit flowing. Solving them is one of life's simple pleasures.

The puzzle masters of Japan create the world's most satisfying puzzles, so Alex Bellos travelled to Tokyo to meet them. These enigmatologists include the god-father of Sudoku, the winner of the WorldPuzzle Championships, an inspiring teacher who uses games to enliven his students' maths lessons, and the puzzle poet whose name has become a Sudoku-solving technique. They use noms de guerre — Edamame, Lenin, Teatime, Sesame Egg — and each has a distinctive style. What unites them are their megawatt brains and the beauty of their hand-crafted puzzles, which will challenge and sharpen your mind.

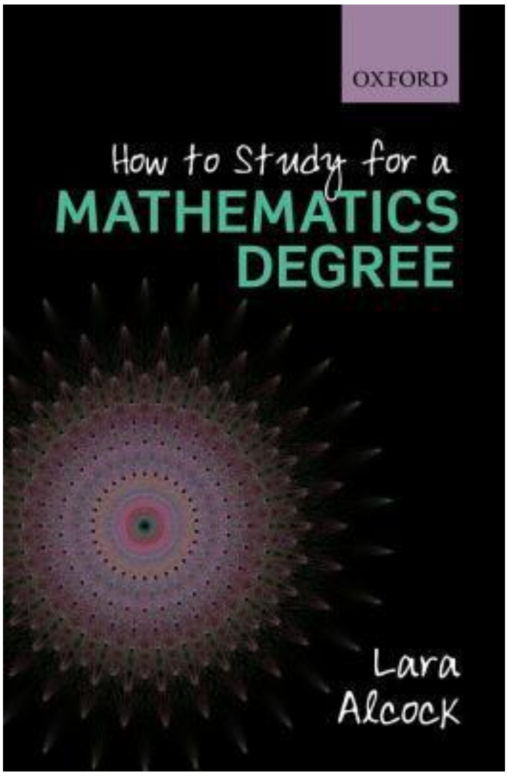
Bellos has collected over 200 of their most ingenious puzzles, rated easy to excruciating , and introduces over 20 new types of addictive problems including Shakashaka and Marupeke.

Arm yourself with pencil, eraser and laser-like focus.

Let's get puzzling . . .

Miscellaneous books

How to Study for a Mathematics Degree (Lara Alcock)



Every year, thousands of students go to university to study mathematics (single honours or combined with another subject). Many of these students are extremely intelligent and hardworking, but even the best will, at some point, struggle with the demands of making the transition to advanced mathematics. Some have difficulty adjusting to independent study and to learning from lectures. Other struggles, however, are more fundamental: the mathematics shifts in focus from calculation to proof, so students are expected to interact with it in different ways. These changes need not be mysterious - mathematics education research has revealed many insights into the adjustments that are necessary - but they are not obvious and they do need explaining.

This no-nonsense book translates these research-based insights into practical advice for a student audience. It covers every aspect of studying for a mathematics degree, from the most abstract intellectual challenges to the everyday business of interacting with lecturers and making good use of study time. Part 1 provides an in-depth discussion of advanced mathematical thinking, and explains how a student will need to adapt and extend their existing skills in order to develop a good understanding of undergraduate mathematics. Part 2 covers study skills as these relate to the demands of a mathematics degree. It suggests practical approaches to learning from lectures and to studying for examinations while also allowing time for a fulfilling all-round university experience.

The first subject-specific guide for students, this friendly, practical text will be essential reading for anyone studying mathematics at university.